Propagation Along a Coaxial Cable with a Helical Shield

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***Abstract*—A leaky coaxial cable is modelled by a dielectric coated conductor shielded by a finite number of unidirectional helical wires. A modal equation is derived and solved numerically for the propagation constants of both the monofilar and bifilar modes. Numerical results are also presented for the effective surface transfer impedance of the shield. This parameter is found to depend, in general, on the propagation constant.**

I. Introduction

The leaky feeder technique [1] is now used to provide radio communication in mine tunnels [2]. Many types of leaky coaxial cables are now available [3], and the surface transfer impedance [4],[5] has been used to characterize the mean electromagnetic properties of braided cable shields. This description has been useful in analyzing propagation along a leaky coaxial cable in a mine tunnel [6]. Such analyses predict the existence of two dominant modes that seem to explain the basic propagation mechanisms quite successfully [7]. The bifilar mode carries most of its energy inside the cable with leakage outside, while the monofilar mode carries most of its energy outside the cable with leakage to the inside of the shield.

Although the surface transfer impedance description and the resultant pair of propagation modes have been useful in describing leaky feeder propagation, the validity and generality of the concept are not well established. Thus it is desirable to perform a boundary value analysis for a specific leaky cable model to test the surface transfer impedance concept. The model we choose consists of a dielectric coated conductor which is shielded by a finite number of unidirectional helices. The transmission-line properties of such a structure have been derived by Casey [8] for the special case where the cable dielectric constant is that of free space. Also, Wait [9] has formulated the case of counterwound helices with an arbitrary dielectric constant for the insulation. Such a structure is an excellent model for a braided coaxial cable, but leads to an infinite system of linear equations; this requires a solution by truncation and numerical inversion. Consequently numerical results have yet to be obtained. Our simpler model of unidirectional helices requires no matrix inversion, but still exhibits the basic features of a leaky cable.

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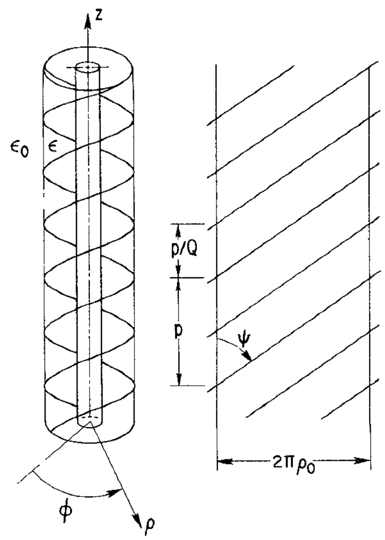

Fig. 1. Perspective view of Q unidirectional helices and planar development of the cylindrical surface (drawn for $Q=2$). The helical wires have radius c.

Also, it is a good model for the less common helical leaky cable which has actually been constructed and tested [10].

II. Formulation

The geometry of the cable model is shown in Fig. 1. The center conductor of radius a is perfectly conducting, and the insulation of permittivity ϵ occupies the region $a<\rho<\rho_0$. The external region is free space with permittivity ϵ_0, and the entire region external to the center conductor and the shield wires has magnetic permeability μ_0. The shield consists of Q equally spaced thin-wire helices, all with the same pitch angle ψ. The helices are defined by the equation

$$\phi=(z/\rho_0)\tan\psi+2\pi q/Q \tag{1}$$

where $q=0,1,\cdots,Q-1$.

The helical wires of radius c are taken to be perfectly conducting for simplicity, but finite wire conductivity could be included by applying an impedance condition at the wires [9].

The model is the same as that of Wait [9] with the counterwound helices removed, and we follow his derivation and notation fairly closely. The time dependence is $\exp(i\omega t)$, and each helix is assumed to carry a current $I_0\exp(-i\beta_0 z)$ where β_0 is the mean propagation constant in the z direction. The assumption of identical current in each helix is a result of ϕ-symmetric excitation. The components of the surface current density in the cylindrical surface $\rho=\rho_0$ are

$$j_z(\phi,z)=I_0\exp(-i\beta_0 z)\frac{\cos\psi}{\rho_0}\sum_{q=0}^{Q-1}\delta\left(\phi-\frac{2\pi z}{p}-\frac{2\pi q}{Q}\right) \quad (2)$$

and

$$j_\phi(\phi,z)=I_0\exp(-i\beta_0 z)\frac{\sin\psi}{\rho_0}\sum_{q=0}^{Q-1}\delta\left(\phi-\frac{2\pi z}{p}-\frac{2\pi q}{Q}\right) \quad (3)$$

where $p(=2\pi\rho_0\cot\psi)$ is the axial period. By using the spectral form of the delta function and performing the q summation analytically [9], the surface current density can be written

$$j_z(\phi,z)=\frac{QI_0\cos\psi}{2\pi\rho_0}\sum_{\substack{l=-\infty\\ n=lQ}}^{\infty}\exp(-i\beta_n z)\exp(in\phi) \quad (4)$$

and

$$j_\phi(\phi,z)=\frac{QI_0\sin\psi}{2\pi\rho_0}\sum_{\substack{l=-\infty\\ n=lQ}}^{\infty}\exp(-i\beta_n z)\exp(in\phi) \quad (5)$$

where $\beta_n=\beta_0+(2\pi n/p)$.

The vector fields $\vec{E}$ and $\vec{H}$ can be expressed in terms of electric and magnetic Hertz vectors with only z components, Π and Π^*. Thus

$$\vec{E}=-i\omega\mu_0\nabla\times(\hat{z}\Pi^*)+(k^2+\nabla\nabla\cdot)(\hat{z}\Pi) \quad (6)$$

and

$$\vec{H}=i\omega\epsilon\nabla\times(\hat{z}\Pi)+(k^2\nabla+\nabla\cdot)(z\Pi^*) \quad (7)$$

where $k=\omega(\mu_0\epsilon)^{1/2}$ is the wavenumber in the region $a<\rho<\rho_0$. In the region $\rho>\rho_0$, we replace ϵ by ϵ_0 and k by $k_0=\omega(\mu_0\epsilon_0)^{1/2}$. From the forms in (4) and (5), we write

$$\Pi=\sum_{\substack{l=-\infty\\ n=lQ}}^{\infty}\Pi_n\exp(-i\beta_n z)\exp(in\phi) \quad (8)$$

and

$$\Pi^*=\sum_{\substack{l=-\infty\\ n=lQ}}^{\infty}\Pi_n^*\exp(-i\beta_n z)\exp(in\phi). \quad (9)$$

The appropriate solutions in the region $\rho>\rho_0$ are

$$\Pi_n=A_nK_n(v_n\rho) \quad (10)$$

and

$$\Pi_n^*=A_n^*K_n(v_n\rho) \quad (11)$$

where $v_n=(\beta_n^2-k_0^2)^{1/2}$, K_n is the modified Bessel function of the second kind of order n, and A_n and A_n^* are constants to be determined. In the region $a<\rho<\rho_0$, the appropriate forms are

$$\Pi_n=B_nZ_n(u_n\rho) \quad (12)$$

where

$$Z_n(u_n\rho)=I_n(u_n\rho)-[I_n(u_na)/K_n(u_na)]K_n(u_n\rho)$$

and

$$\Pi_n^*=B_n^*Z_n^*(u_n\rho) \quad (13)$$

where

$$Z_n^*(u_n\rho)=I_n(u_n\rho)-[I_n'(u_na)/K_n'(u_na)]K_n(u_n\rho).$$

I_n is the modified Bessel function of the first kind, $u_n=(\beta_n^2-k^2)^{1/2}$, and B_n and B_n^* are constants to be determined.

The boundary conditions at the shield are that the tangential electric fields are continuous and that the tangential magnetic fields are discontinuous by the amount of the surface current. That is,

$$E_z(\rho_0^-)=E_z(\rho_0^+) \quad (14)$$

$$H_z(\rho_0^-)=H_z(\rho_0^+)+j_\phi(\phi,z) \quad (15)$$

$$E_\phi(\rho_0^-)=E_\phi(\rho_0^+) \quad (16)$$

$$H_\phi(\rho_0^-)=H_\phi(\rho_0^+)-j_z(\phi,z). \quad (17)$$

Simultaneous solution of (14)–(17) leads to the following expressions for the coefficients A_n, B_n, A_n^*, and B_n^* [9]

$$A_n=\left[i\omega\mu_0v\left(\frac{v}{u}\frac{Z^{*\prime}}{Z^*}K-K'\right)\left(\frac{n\beta}{u^2\rho_0}\sin\psi-\cos\psi\right)-\frac{n\beta}{\rho_0}K\left(\frac{v^2}{u^2}-1\right)\frac{i\omega\mu m_0}{u}\frac{Z^{*\prime}}{Z^*}\sin\psi\right]\frac{QI_0}{2\pi\rho_0D} \quad (18)$$

$$A_n^*=\left[\frac{k_0^2v}{n}\frac{Z^{*\prime}}{Z^*}\left(\frac{\epsilon}{\epsilon_0}\frac{v}{u}\frac{Z'}{Z}K-K'\right)\sin\psi-\left(\frac{n\beta}{u^2\rho_0}\sin\psi-\cos\psi\right)\frac{n\beta}{\rho_0}K\left(\frac{v^2}{u^2}-1\right)\right]\frac{QI_0}{2\pi\rho_0D} \quad (19)$$

$$B_n=A_nv^2K/(u^2Z) \quad (20)$$

$$B_n^*=\left(v^2KA^*-\frac{QI_0}{2\pi\rho_0}\right)\Big/(u^2Z) \quad (21)$$

where

$$D=k_0^2v^2\left[\frac{v}{u}\frac{Z^{*\prime}}{Z^*}K-K'\right]\left[\frac{\epsilon}{\epsilon_0}\frac{v}{u}\frac{Z'}{Z}K-K'\right]-\left(\frac{n\beta}{\rho_0}K\right)^2\left(\frac{v^2}{u^2}-1\right)^2.$$

Also for convenience we have written $u=u_n$, $v=v_n$, $Z=Z_n(u_n\rho_0)$, $Z'=Z_n'(u_n\rho_0)$, $Z^*=Z_n^*(u_n\rho_0)$, $Z^{*\prime}=Z_n^{*\prime}(u_n\rho_0)$, $K=K_n(v_n\rho_0)$, $K'=K'(v_n\rho_0)$, *and* $\beta=\beta_n$.

The tangential electric fields in the external region, $\rho>\rho_0$, are given by

$$E_\phi=\sum_{\substack{l=-\infty\\ n=lQ}}^{\infty}\left[i\omega\mu_0v_nA_n^*K_n'(v_n\rho)+\frac{n\beta_n}{\rho}A_nK_n(v_n\rho)\right]\cdot\exp(in\phi)\exp(-i\beta_nz) \quad (22)$$

and

$$E_z = -\sum_{\substack{l=-\infty \\ n=lQ}}^{\infty} v_n^2 A_n K_n(v_n\rho)\exp(in\phi)\exp(-i\beta_n z). \tag{23}$$

A_n and A_n^* can be expressed directly in terms of the helix current I_0

$$A_n = P_n I_0$$

and

$$A_n^* = P_n^* I_0 \tag{24}$$

where P_n and P_n^* are defined by (18) and (19).

III. Application of the Thin-Wire Condition

Since the shield wires are assumed to be very thin, the longitudinal electric field is essentially uniform around the wire circumference. Thus the boundary condition that the tangential electric field is zero on the shield wires can be applied to any convenient point on the wires. Also, due to the symmetry, it is sufficient to apply the condition at only one of the Q wires. Wait [9] and Casey [8] chose to apply the condition at the top of the $q=0$ wire which is defined by the spiral $z=(p/2\pi)\phi + c/\sin\psi, \rho=\rho_0$. However, convergence of the numerical solution is better if the boundary condition is applied on the outside of $q=0$ wire which is defined by the spiral $z=(p/2\pi)\phi$, $\rho=\rho_0+c$. Thus we use this boundary condition which is written

$$E_z\cos\psi + E_\phi \sin\psi + E_z^p\cos\psi = 0 \tag{25}$$

where E_z^p is the axial component of the primary field. It is the field which would exist at the surface $\rho=\rho_0+c$ for the same cylindrical structure in the absence of the shield wires. The actual expression for E_z^p is not required here, but it has been derived for the case of plane wave excitation by Wait [9].

By substituting (22)–(24) ino (25), the thin-wire boundary condition becomes

$$QI_0 \sum_{\substack{l=-\infty \\ n=lQ}}^{\infty} R_n + E_z^p\cos\psi = 0 \tag{26}$$

where

$$R_n = \left(-v_n^2\cos\psi + \frac{n\beta_n}{\rho_0+c}\sin\psi\right)P_n K_n[v_n(\rho_0+c)] + i\omega\mu_0 v_n \sin\psi P_n^* K_n'[v_n(\rho_0+c)]. \tag{27}$$

Thus I_0 is now known if E_z^p is specified. The result in (26) can be shown to agree quite closely with that of Casey for the special case of $\epsilon=\epsilon_0$, although the notation is somewhat different. The reason for choosing the match point on the outside of the wire can be seen by examining the convergence of the n summation in (26). By employing the uniform asymptotic expressions for the modified Bessel function [11] in (27), R_n can be shown to decay exponentially for large n:

$$R_n \sim b(n)\exp(-nc\sin\psi\tan\psi/\rho_0) \tag{28}$$

where $b(n)$ is an algebraic function of n. This exponential decay speeds the convergence of the n summation in (26) which in general must be evaluated numerically. When the match point is taken on the tops of the wire [8],[9], R_n has only an algebraic decay for large n. However, Latham [12] has performed the required summation analytically for some special cases.

IV. Modal Equation

The modal equation, obtained by setting the primary field equal to zero in (26), is

$$\sum_{\substack{l=-\infty \\ n=lQ}}^{\infty} R_n = 0 \tag{29}$$

where R_n is given by (27). In general, (29) must be solved numerically for the unknown propagation constant β_0, but there are some special cases where the mode equation simplifies considerably.

We consider first the special case $\epsilon=\epsilon_0$. If, in addition, the relevant cable dimensions are electrically small ($k_0\rho_0$ and $k_0 p \ll 1$), the following approximate solution for β_0 is obtained:

$$\frac{\beta_0}{k_0} \cong \left\{\frac{\ln(\rho_0/a) + \tan^2\psi\left[\left(1-\frac{a^2}{\rho_0^2}\right)\Big/2 - 2S_2\right]}{\ln(\rho_0/a)+2S_1}\right\}^{1/2} \tag{30}$$

where

$$S_1 = \sum_{\substack{l=1 \\ n=lQ}}^{\infty}\left[I_n(n\tan\psi) + \frac{I_n(n\tan\psi a/\rho_0)}{K_n(n\tan\psi a/\rho_0)}K_n(n\tan\psi)\right] \cdot K_n\left[n\tan\psi\left(1+\frac{c}{\rho_0}\right)\right] \tag{31}$$

and

$$S_2 = \sum_{\substack{l=1 \\ n=2Q}}^{\infty}\left[I_n'(n\tan\psi) + \frac{I_n'(n\tan\psi a/\rho_0)}{K_n'(n\tan\psi a/\rho_0)}K_n'(n\tan\psi)\right] \cdot K_n'\left[n\tan\psi\left(1+\frac{c}{\rho_0}\right)\right]. \tag{32}$$

The result in (30) is in close agreement with the propagation constant obtained from Casey's [8] solution, although he does not specifically deal with the modal equation. The value of β_0 obtained from (30) is real and greater than k_0. This is the type of slow-wave solution which is to be expected with such a helical structure [13]. If the number of wires Q becomes large, then S_1 and S_2 become small and a remarkably simple expression for β_0 results:

$$\frac{\beta_0}{k_0} \cong \left[1 + \frac{\left(1-\frac{a^2}{\rho_0^2}\right)\tan^2\psi}{2\ln(\rho_0/a)}\right]^{1/2}. \tag{33}$$

Here we note that, as ψ tends to zero, β_0/k_0 approaches one and we have the expected TEM transmission-line mode. On the other hand, as ψ increases, β_0/k_0 becomes greater than one and the mode is progressively slowed down.

For $\epsilon > \epsilon_0$, the mode equation is more complicated. However, for Q large and $k\rho_0$ and kp small, a solution of the following form is found

$$\frac{\beta_0}{k_0} \cong \left(\frac{\epsilon}{\epsilon_0}\right)^{1/2} \left[1 + \frac{\left(1 - \frac{a^2}{\rho_0^2}\right)\tan\psi}{2\ln(\rho_0/a)} \right]^{1/2}. \tag{34}$$

For ψ approaching zero, β_0/k_0 approaches $(\epsilon/\epsilon_0)^{1/2}$ and we clearly have the bifilar mode. For increasing ψ this mode also becomes slower. Another mode is also found numerically in the vicinity $\beta_0/k_0 \cong 1$, but we could find no analytical solution. This mode is clearly the monofilar mode which has the character of a Goubau wave since no return current path is available for the cable in free space.

V. Surface Transfer Impedance

The surface transfer impedance Z_T is defined as the ratio of averaged axial electric field at the shield divided by the averaged axial shield current (proportional to the discontinuity in H_ϕ)

$$Z_T = \frac{\bar{E}_z|_{\rho=\rho_0}}{2\pi\rho_0\left[\bar{H}_\phi|_{\rho=\rho_0^+} - \bar{H}_\phi|_{\rho=\rho_0^-}\right]}. \tag{35}$$

The bar indicates that the averaging is carried out over z (from 0 to p) and ϕ (from 0 to 2π). When this averaging is carried out, only the $n=0$ harmonics remain and Z_T is given by

$$Z_T = \frac{E_z^p - v_0^2 A_0 K_0(v_0\rho_0)}{i\omega 2\pi\rho_0\left\{\epsilon u_0 B_0\left[I_0'(u_0\rho_0) - \frac{I_0(u_0 a)}{K_0(U_0 a)}K_0'(u_0\rho_0)\right] - \epsilon_0 v_0 A_0 K_0'(v_0\rho_0)\right\}}. \tag{36}$$

For the usual case where $u_0\rho_0$ and $v_0\rho_0$ are small, the small argument approximations for the modified Bessel functions can be used to simplify (36)

$$Z_T \cong \frac{E_z^p + v_0^2 A_0 \ln(v_0\rho_0)}{i\omega 2\pi\left[\epsilon_0 A_0 - \epsilon B_0/\ln(u_0 a)\right]}. \tag{37}$$

For the special case where the modal equation (29) is satisfied, we can set E_z^p equal to zero. If we also substitute (20) into (37), Z_T simplifies to

$$Z_T \cong \frac{v_0^2 \ln(v_0\rho_0)}{i\omega 2\pi\left[\epsilon_0 - \epsilon\frac{v_0^2\ln(v_0\rho_0)}{u_0^2\ln(\rho_0/a)}\right]}. \tag{38}$$

For the further special case of $\epsilon = \epsilon_0$, (38) simplifies to

$$Z_T = \frac{i\omega u_0 \ln(\rho_0/a)}{2\pi}\left(\frac{\beta_0^2}{k_0^2} - 1\right). \tag{39}$$

This form agrees exactly with Casey's result [8] for the case where the modal equation is satisfied. It should be stressed that (38) and (39) cannot be used to determine the dependence of Z_T on β_0 because (38) and (39) are only valid for those values of β_0 where the modal equation is satisfied.

In order to gain some insight into the dependence of Z_T on pitch angle ψ, we can examine the case for large Q. If the solution of β_0 in (33) is substituted into (39), then Z_T becomes

$$Z_T \cong \frac{i\omega u_0}{4\pi}\left(1 - \frac{a^2}{\rho_0^2}\right)\tan^2\psi. \tag{40}$$

Note that as ψ increases from 0 to $\pi/2$, $|Z_T|$ increases from zero to infinity. Thus the cable becomes more leaky as ψ is increased. The optical coverage C of the shield is simply given by

$$V = \frac{Qc}{\pi\rho_0\cos\psi}. \tag{41}$$

From (40) and (41), we see that as ψ increases, the optical coverage increases, but the cable becomes more leaky (larger Z_T). This points up the well-known fact that optical coverage is not a good measure of cable shielding for most cables [14].

In many cases the surface transfer impedance is positive imaginary and proportional to frequency. This has lead to the following definition for surface transfer inductance [4]:

$$L_T = Z_T/(i\omega). \tag{42}$$

For this special result given in (40), this definition yields a positive real value of L_T which is independent of frequency. For the more general result of (36), L_T will actually depend on both the frequency and the propagation constant.

VI. Numerical Results

A computer program was written to solve the general modal equation (29). Since the structure is lossless for ϵ real, we are interested in real values of β_0 greater than k_0 such that the fields decay for large ρ. The bisection method [15] was used to solve (29), and Z_T was calculated using the general expression in (36).

All results were computed for the following parameters: $a = 1.5$ mm, $\rho_0 = 10$ mm, and $c = 0.5$ mm. In Figs. 2–5, we show results for the special case $\epsilon = \epsilon_0$, and here only one mode was found.

In Fig. 2, the propagation constant β_0 is shown for various values of Q as a function of pitch angle ψ for a frequency of 10 MHz. As expected, β_0 approaches k_0 for small ψ, and β_0 decreases as the number of shield wires Q is increased. The dashed curve for large Q is obtained from (33). In Fig. 3, the surface transfer inductance,

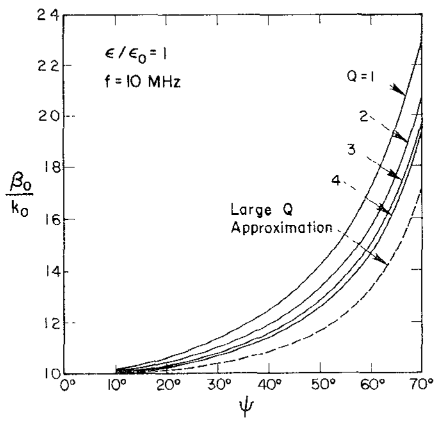

Fig. 2. Relative propagation constant β_0/k_0 as a function of pitch angle ψ for an air-filled cable at a frequency of 10 MHz.

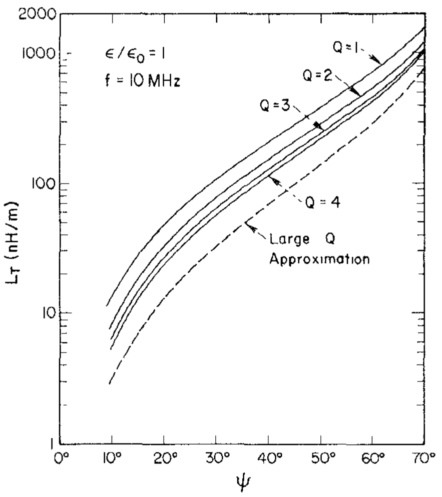

Fig. 3. Surface transfer inductance as a function of pitch angle.

$L_T = Z_T/(i\omega)$ (in nanohenrys/meter), is also shown as a function of ψ.

The frequency dependence of the propagation constant is shown in Fig. 4 for $\psi = 30°$. For sufficiently low frequencies, β_0/k_0 is essentially independent of frequency as predicted by the approximate result in (30), but this idealization gradually fails as the frequency is increased. A similar effect is observed for the transfer inductance in Fig. 5.

In Figs. 6–9, results are shown for the case of a dielectric insulation, where $\epsilon/\epsilon_0 = 2.5$. The propagation constant of the bifilar mode is shown in Fig. 6, and $\beta_0 \simeq k$ for small ψ and large Q as expected. The propagation constant of the monofilar mode is shown in Fig. 7, and β_0 approaches k_0 for small ψ and large Q.

The surface transfer impedance for the bifilar mode is shown in Fig. 8. Z_T is pure imaginary, but the imaginary part is actually negative for small values of ψ. In such cases, a surface transfer inductance is obviously not an adequate characterization of the shield. The surface transfer impedance for the monofilar mode is shown in Fig. 9, and there are substantial differences from the bifilar mode results of Fig. 8. Such differences are to be expected since

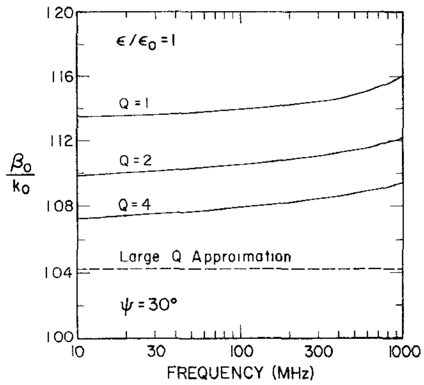

Fig. 4. Relative propagation constant as a function of frequency for an air-filled cable.

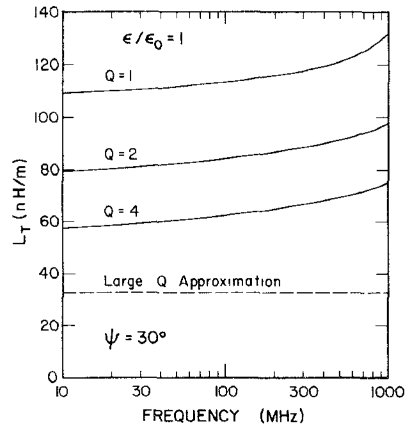

Fig. 5. Surface transfer inductance as a function of frequency for an air-filled cable.

Z_T is known to depend on the propagation constant β_0 [8],[16].

VII. Concluding Remarks

An idealized leaky coaxial cable has been modelled by a dielectric coated conductor shielded by a finite number of unidirectional helical wires. The rigorous modal equation was solved numerically for propagation constants of the bifilar and monofilar modes. As expected, the propagation constant of the bifilar mode is close to the wave number of the insulation, and the propagation constant of the monofilar mode is slightly greater than that of free space. Since there is no return current path for the isolated cable in free space, the monofilar mode takes on the character of a Goubau mode. For the special case of an air-filled cable, only one propagation mode is found.

The surface transfer impedance has been calculated and is generally found to increase as the pitch angle of the shield wires is increased. The optical coverage (or relative metal area) also increases as the pitch angle increases; this is not a good measure of cable shielding. The surface transfer impedance cannot be represented simply by a transfer inductance which is independent of frequency and propagation constant.

A useful extension to this work would be a numerical study of the counterwound helical model formulated by

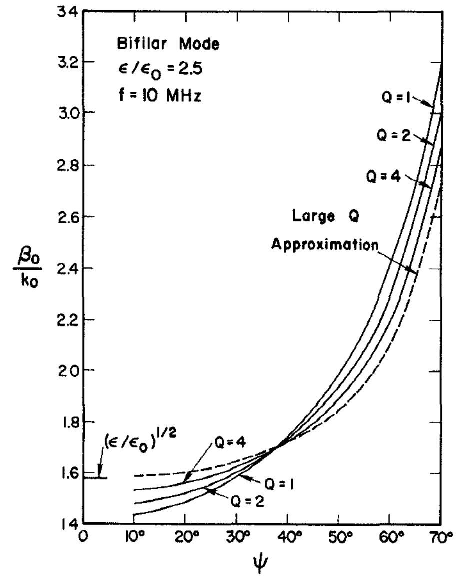

Fig. 6. Relative propagation constant of the bifilar mode of a dielectric-filled cable as a function of pitch angle.

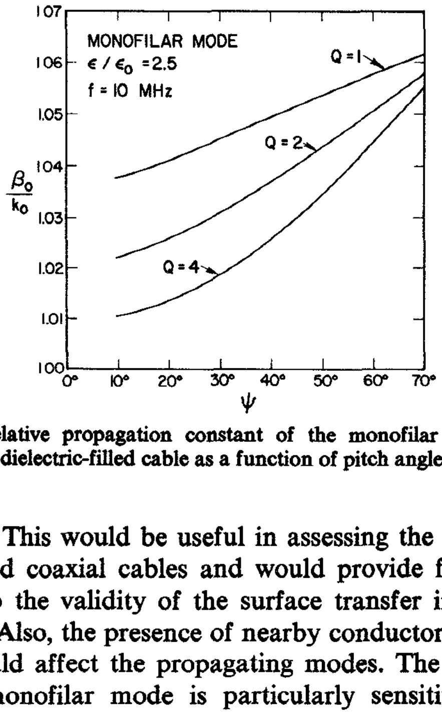

Fig. 7. Relative propagation constant of the monofilar mode of a dielectric-filled cable as a function of pitch angle.

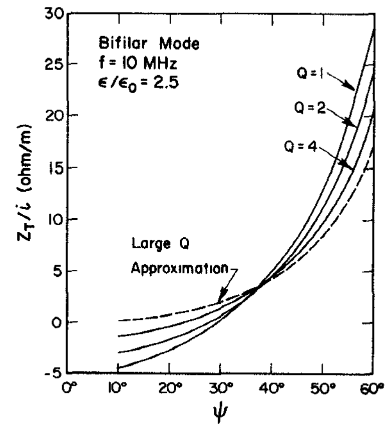

Fig. 8. Surface transfer impedance of the bifilar mode as a function of pitch angle.

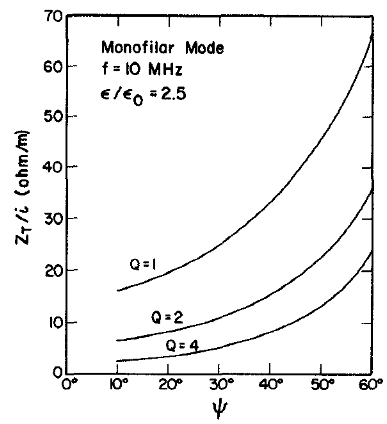

Fig. 9. Surface transfer impedance of the monofilar mode as a function of pitch angle.

Wait [9]. This would be useful in assessing the properties of braided coaxial cables and would provide further insight into the validity of the surface transfer impedance concept. Also, the presence of nearby conductors or interfaces could affect the propagating modes. The character of the monofilar mode is particularly sensitive to the separation between the cable and a nearby interface [17], [18].

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